GCSE Maths – Algebra

Laws of Indices

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving indices. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Simplify $2a^5 \times 6b^2 \times 3a^7$

Step 1: Write out the expression by separating any constants that are present and grouping these together. Then group any variables that are present together, (grouping the same letters together).

$$2a^5 \times 6b^2 \times 3a^7 = 2 \times 3 \times 6 \times a^5 \times a^7 \times b^2$$

Step 2: Multiply together any constants that are present.

$$2a^5 \times 6b^2 \times 3a^7 = 2 \times 3 \times 6 \times a^5 \times a^7 \times b^2 = 36 \times a^5 \times a^7 \times b^2$$

Step 3: Multiply any terms with powers together - if they have the same base, add the powers.

In this example we add the powers of 'a' as they have the same base:

$$a^5 \times a^7 = a^{(5+7)} = a^{12}$$

$$36 \times a^5 \times a^7 \times b^2 = 36a^{(5+7)} \times b^2 = 36a^{12} \times b^2 = 36a^{12}b^2$$

Answer: $\mathbf{36a^{12}b^2}$

Guided Example

Simplify $12c^3 \times 3c^7 \times 5d^2$

Step 1: Write out the expression by separating any numbers that are present and grouping these together. Then group any variables that are present together, (grouping the same letters together).

$$12 \times 3 \times 5 \times c^3 \times c^7 \times d^2$$

Step 2: Multiply together any constants that are present.

$$180 \times c^7 \times c^3 \times d^2$$

Step 3: Multiply any terms with powers together - if they have the same base, add the powers

apply indices rule

$$180 \times c^{(7+3)} \times d^2 = 180c^{10}d^2$$

write in alphabetical order.

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Simplify the following:

a) $8p^4 \times 4p^8$

separate terms

$= 8 \times 4 \times p^4 \times p^8$

multiply constants

$= 32 \times p^4 \times p^8$

add powers

$= 32 \times p^{(4+8)}$

simplify

$= \boxed{32p^{12}}$

b) $7r^6 \times 8s^5 \times 9r^4$

$= 7 \times 8 \times 9 \times r^6 \times s^5 \times r^4$

$= 504 \times r^6 \times r^4 \times s^5$

$= 504 \times r^{6+4} \times s^5$

$= \boxed{504r^{10}s^5}$

c) $\underbrace{2^5 \times 2^8}_{\text{these are constants because they do not contain an algebraic term}} \times a^3$

indices rules still apply to constants!

$= 2^{5+8} \times a^3$

$= 2^{13} \times a^3$

$= \boxed{8192a^3}$

d) $9f^4 \times 4^8 \times 2g^8 \times 4^5$

$= 4^8 \times 4^5 \times 2 \times 9 \times f^4 \times g^8$

$= 4^{8+5} \times 2 \times 9 \times f^4 \times g^8$

$= 4^{13} \times 2 \times 9 \times f^4 \times g^8$

$= \boxed{1207959552f^4g^8}$

e) $5^5 \times e^4 \times 4 \times 5^5 \times 5e^3$

$= 5^5 \times 5^5 \times 5 \times 4 \times e^4 \times e^3$

$= 5^{5+5+1} \times 4 \times e^{4+3}$

$= 5^{11} \times 4 \times e^7$

$= \boxed{195312500e^7}$

Section B

Worked Example

Simplify $8a^8b^4 \div 4a^3b$

Step 1: The expression can be written as a fraction. This might help you to visualise the common terms.

$$8a^8b^4 \div 4a^3b = \frac{8a^8b^4}{4a^3b}$$

Step 2: Divide any common constants to simplify the fraction.

$$8a^8b^4 \div 4a^3b = \frac{8a^8b^4}{4a^3b} = \frac{2a^8b^4}{a^3b}$$

In this example, dividing the top and bottom of the fraction by 4 gives a more simplified fraction.

Step 3: Divide any common terms which have powers - if they have the same base, subtract the powers.

In this example we subtract the powers of $'a'$ and $'b'$ as they have the same base:

$$a^8 \div a^3 = a^{(8-3)} = a^5$$
$$b^4 \div b^1 = b^3$$

$$\frac{2a^8b^4}{a^3b^1} = 2a^{(8-3)}b^{(4-1)} = 2a^5b^3$$

Answer: $\mathbf{2a^5b^3}$

Guided Example

Simplify: $9c^5d^2 \div 3cd$

Step 1: Sometimes the expression can be written as a fraction with a numerator and a denominator. This might help you to visualise the common terms.

$$\frac{9c^5d^2}{3cd}$$

Step 2: Divide any common constants to simplify the fraction.

$$9 \div 3 = 3 \quad \frac{\cancel{9}c^5d^2}{\cancel{3}cd} = \frac{3c^5d^2}{cd}$$

Step 3: Divide any common terms which have powers - if they have the same base, subtract the powers.

$$c^5 \div c^1 = c^{5-1} = c^4$$

$$\frac{3c^5d^2}{cd} = \boxed{3c^4d}$$

$$d^2 \div d = d^{2-1} = d$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Simplify the following:

a) $x^2y^3 \div xy^2$

rewrite as fraction

apply indices rule

$$\frac{x^2y^3}{xy^2} = x^{2-1}y^{3-2} = \boxed{xy}$$

b) $16f^7g^2 \div 4f^3g$

divide constant first: $16 \div 4 = 4$

$$\frac{16f^7g^2}{4f^3g} = \frac{4f^7g^2}{f^3g} = 4f^{7-3}g^{2-1} = \boxed{4f^4g}$$

c) $2r^8s^5t^2 \div r^2s^2$

no t in denominator so this stays the same

$$\frac{2r^8s^5t^2}{r^2s^2} = 2r^{8-2}s^{5-2}t^2 = \boxed{2r^6s^3t^2}$$

d) $21j^8k^3l^3 \div 3k^2l$

$$\frac{21j^8k^3l^3}{3k^2l} = \frac{7j^8k^3l^3}{k^2l} = 7j^8k^{3-2}l^{3-1} = \boxed{7j^8kl^2}$$

e) $45x^9y^{10}z^5 \div 5x^{12}y^7$

this is the same as writing $\frac{1}{x^3}$

$$\frac{45x^9y^{10}z^5}{5x^{12}y^7} = \frac{9x^9y^{10}z^5}{x^{12}y^7} = 9x^{9-12}y^{10-7}z^5 = \boxed{9x^{-3}y^3z^5}$$

Section C

Worked Example

Simplify $(8^2 a^7)^5$

Step 1: When raising one power to another, multiply the powers together. If there is a complicated term within the bracket, separate the components and deal with them individually.

$$(8^2a^7)^5 = (8^2)^5 \times (a^7)^5$$

$$(8^2)^5 = 8^{2\times5} = 8^{10}$$
$$(a^7)^5 = a^{7\times5} = a^{35}$$

$$\mathbf{(8a^7)^5 = 8^{10}a^{35}}$$

Step 2: Calculate the value of any constant that is raised to a numerical power.

$$(8a^7)^5 = 8^{10}a^{35} = 8^{10} \times a^{35} = 1073741824a^{35}$$

In this example it is probably best to expand the 8^{10} *using a calculator.*

For high powers, the number raised to a power can be left in base-index form.

Answer: $\mathbf{1073741284a^{35}}$ ***or*** $\mathbf{8^{10}a^{35}}$

Guided Example

Simplify $(17b^2)^3$

Step 1: When raising one power to another, multiply them together. If there is a complicated term within the bracket, separate the components and deal with them individually.

apply indices rule

$$(17b^2)^3 = (17 \times b^2)^3 = 17^3 \times b^{2\times3} = 17^3 \times b^6$$

Step 2: Calculate the value of any constant that is raised to a numerical power.

$$17^3 \times b^6 = 4913b^6$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Simplify the following:

a) $(8^9)^3$ $= 8^{9\times 3}$

$= 8^{27}$

This is a high power so leave in base-index form

b) $(h^4)^{16}$ $= h^{4\times 16}$

$= h^{64}$

deal with each term separately.

c) $(ft^8)^5$ $= (f \times t^8)^5$

$= f^5 \times t^{8\times 5}$

$= f^5 \times t^{40}$

$= f^5 t^{40}$

d) $(c^2d^3)^4$ $= (c^2 \times d^3)^4$

$= c^{2\times 4} \times d^{3\times 4}$

$= c^8 \times d^{12}$

$= c^8 d^{12}$

e) $(17^5z^9)^{16}$ $= (17^5 \times z^9)^{16}$

$= 17^{5\times 16} \times z^{9\times 16}$

$= 17^{80} \times z^{144}$

$= 17^{80} z^{144}$

leave as a power